RANGE OF VALIDITY OF FIELD EMISSION EQUATIONS

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The analytical expressions for the current density *J* and total energy distribution (TED) can be derived for electron field emission over a limited range of temperature *T*, electric field *F* and work function *φ*. The analytical expressions for *J* are:

$$J_{CFE} = J_{FN} \frac{\pi p}{\sin(\pi p)} \qquad\qquad J_{ES} = J_{SE} \frac{\pi q}{\sin(\pi q)}$$

where *p* and *q* are dimensionless parameters with $p(F,T,\varphi)=kT/d$ and $q(F,T)=C_3F^{3/4}/kT$ and J_{FN} and J_{SE} are the well known Fowler-Nordheim and Schottky equations. The analytical expressions for the total energy distribution (TED) *J(E)* are:

$$TED_{CFE} = C_1 \frac{\exp(E/d)}{1+\exp(E/pd)} \qquad\qquad TED_{ES} = C_2 \frac{\ln[1+\exp(E/qkT)]}{1+\exp[(E+\varphi-E_S)/kT]}$$

where $d=heF/4\pi(2m\varphi)^{1/2}$ and $E_S = (e^3F/4\pi\varepsilon_o)^{1/2}$ and C_1 and C_2 are constants. Previous studies[1,2] have examined the range *F* and *T* over which these relationships are valid for the CFE (cold field) or ES (extended Schottky) emission regimes. These relationships have been assumed to be reasonably valid for $p<0.7$ and $q<0.7$.

This investigation determines the valid ranges by comparison of the analytical expression to a rigorous numerical calculation based on fundamental principles. We specifically focus on the range of values for *p* and *q* to find where the above equations compare favorably to the rigorous numerical calculation[3]. Figs.1 and 2 show the ratios of the analytically to the numerically calculated values of the current density *J* and full width at half maximum (FWHM) of the TED for a low (2.5 eV) value of *φ*. From these figures one can see that the CFE analytical expressions for *J* and the FWHM of the TED are accurate in the range of $p< 0.7$ as expected. Similarly the ES analytical expression for the TED compares favorably with the numerical expression for $q<0.76$. Surprisingly, the analytical expression for *J* deviates substantially from the numerical results for $q>0.2$.

A substantial discrepancy between *J(A)* and *J(N)* in the ES region, as shown in Fig. 3, holds for all values of *φ* and suggests caution should be exercised when using J_{ES} to extract emission parameters, such as *φ* and $\beta = F/V$, from experimental *I(V)* data. The valid regions of *p* and *q* can be used to find valid ranges for *T*, *F* and *φ*. Fig. 3 shows the temperature range over which the analytical expressions are valid for φ =2.5 eV. It is also clear there is a temperature range for which only numerical solutions are available for *J* and *J(E)* Further comparison between the analytical and numerical values for *J* and *J(E)* at other values of *T*, *F* and *φ* will be given to determine valid ranges for each.

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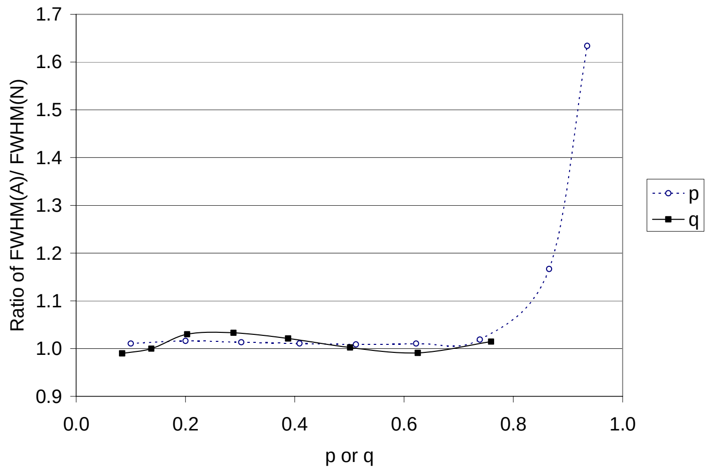

Fig. 1 Curves show the ratio of the analytical (A) *J* to the numerical (N) *J* as a function of *p* and *q* at *φ=2.5* eV. Ideally, the ratio would be 1.0.

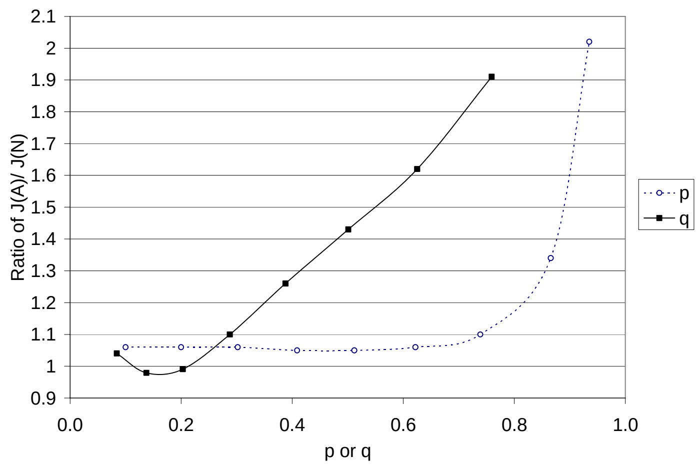

Fig. 2 Curves show the ratio of the analytical (A) *FWHM* to the numerical (N) *FWHM* as a function of *p* and *q* at *φ=2.5* eV. Ideally, the ratio would be 1.0.

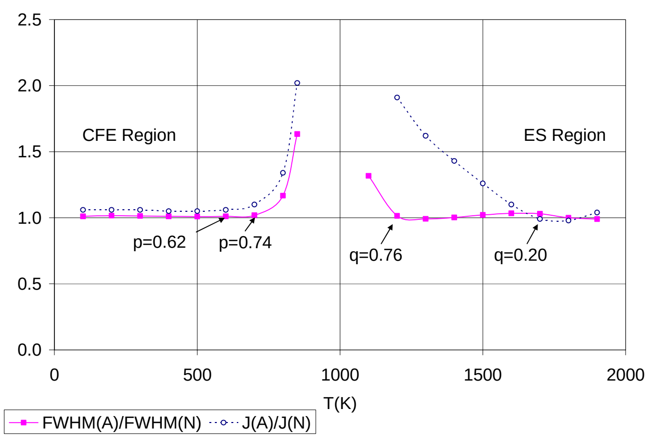

Fig. 3 Curves show the ratio of the analytical (A) to the numerical (N) for *J* and *FWHM* of the TED at *φ* = 2.5 eV. All curves are calculated at $J=1\text{x}10^9$ A/m^2.

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[1] M. Fransen, J. Faber, T. van Rooy, P. Timeijer and P. Kruit, J. Vac. Sci. Technol. **B16(4),** 2063 (1998)
[2] A. El-Kareh, J. C. Wolfe, and J. E. Wolfe, J. Appl. Phys. 48, 4749 (1977)
[3] A. Modinos, *Field, Thermionic and Secondary Electron Emission Spectroscopy*, Springer, 1984 (ISBN 978-0306413213)