Iterative Phase Recovery Using Wavelet Domain Constraints

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Phase retrieval is a central problem in coherent xray diffraction microscopy. Various methods have been proposed to solve the problem with the most successful being iterative methods such as Fienup's Hybrid Input Output algorithm and the generalized Difference Map algorithm [1, 2]. These algorithms are constraint satisfaction problems where a solution satisfying two sets of constraints is sought. In the context of diffraction microscopy these are diffraction measurements in the Fourier domain and typically some form of finite support in the spatial domain.

In many cases much of the available *a priori* information is not utilized by the algorithm. A low resolution SEM image may be used to replace diffraction information lost by a beam stop and to provide an initial support [3]. Information more detailed than a support constraint is not easily described in the spatial domain but can be expressed in the wavelet domain.

Wavelets have the advantage of being spatially localized and are more able to describe non-stationary signals. A low resolution version of the sample provides low order wavelet coefficients. These can be enforced while the higher order coefficients can adapt to match the detailed information from the diffraction measurements. We gain the advantage of utilizing much more of the *a priori* information. Figure 1 illustrates a successful reconstruction.

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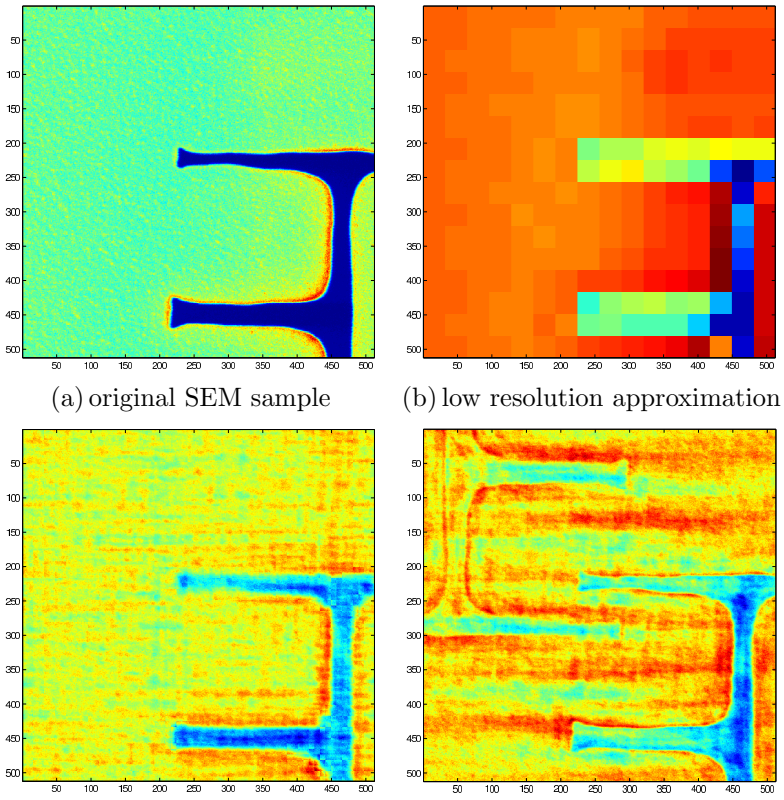

(a) original SEM sample (b) low resolution approximation

(c) reconstruction (d) reconstruction from spatial support

FIG. 1: Iterative phase reconstruction where constraints are enforced in the Fourier domain and in the wavelet domain. (a) shows the original sample from which diffraction measurements are obtained (b) shows the low resolution approximation used for the wavelet constraint (c) shows the reconstruction using constraints in wavelet space while (d) shows a reconstruction using a basic finite support constraint (the support is not modified as the algorithm progresses—as such the phase ambiguity is not resolved).

[1] Veit Elser. Phase retrieval by iterated projections. *Journal of the Optical Society of America*, 20(1):40–55, 2003.

[2] J. R. Fienup. Phase retrieval algorithms: a comparison. *Applied Optics*, 21:2758–2769, 1982.

[3] Jianwei Miao, Tetsuya Ishikawa, Bart Johnson, Erik H. Anderson, Barry Lai, and Keith O. Hodgson. High resolution 3d x-ray diffraction microscopy. *Phys. Rev. Lett.*, 89(8):088303, Aug 2002.