# ACCURATE CALCULATION OF THE FIELD FACTOR FOR THE SCHOTTKY CATHODE

A. Bahm, G.A. Schwind, L.W. Swanson

FEI Co. Hillsboro, OR 97124

Studies of field emission type emitters typically use the field enhancement factor Beta ($\beta$) to characterize the geometry shape of the end form and to monitor changes in the end form. In addition, the beta factor is a critical parameter in many emission calculations when used to obtain the applied field from the experimentally accessible parameter of extraction voltage. Therefore it is important to be able to calculate $\beta$ accurately. The authors present a method which calculates beta with higher accuracy and reliability than currently used methods. Most methods are based on the analytical equations which have been shown to have a limited range of validity which leads to poor estimation of the beta factor.

The Schottky $J_s$ and extended Schottky $J_{ES}$ current density from a Schottky emitter with a field *F* and work function $\varphi$ is given by

$$J_s = \frac{4\pi m_e (kT)^2}{h^3} \exp\left( \frac{e^{\frac{3}{2}} F^{\frac{1}{2}}}{(4\pi\varepsilon_o)^{\frac{1}{2}}} - \frac{\varphi}{kT} \right) \qquad J_{ES} = J_S \frac{\pi q}{\sin(\pi q)}$$

in the Schottky and extended Schottky regimes respectively where $q = \frac{h(4\pi\varepsilon_o e)^{1/4} F^{3/4}}{2\pi^2 {m_e}^{1/2} kT}$, a dimensionless parameter, is a measure of the tunneling vs. thermionic current. *F* and *J* are determined by the experimentally accessible parameters of extraction voltage *V* and angular current density *I'* and the relations $F = \beta V$ and $I' = J\left(\frac{r}{m}\right)^2$ where *r* and *m* are the apex emitter radius and angular magnification respectively[1].

The geometric factor $\beta$ was originally determined from a linear fit to $\ln(J_S)$ vs. $\sqrt{F}$. However, in the extended Schottky regime, the additional $\pi q/sin(\pi q)$ term makes this inaccurate (see Figure 1) even when an iterative fitting algorithm is used[1]. Recent evaluation of the range of validity of the Schottky equation has shown for *q* > 0.3 that the analytical equation for $J_{ES}$ is a poor approximation compared to numerical simulation[2] (see Figure 2).

An improved method for accurately determining $\beta$, using the numerical simulation and a Nelder-Mead iterative fitting algorithm was written. This investigation will show that the a program that fits the experimental *I'(V)* data to the numerical solution robustly returns an accurate value for $\beta$, even when the ratio (*r*/*m*) was varied over a wide range.

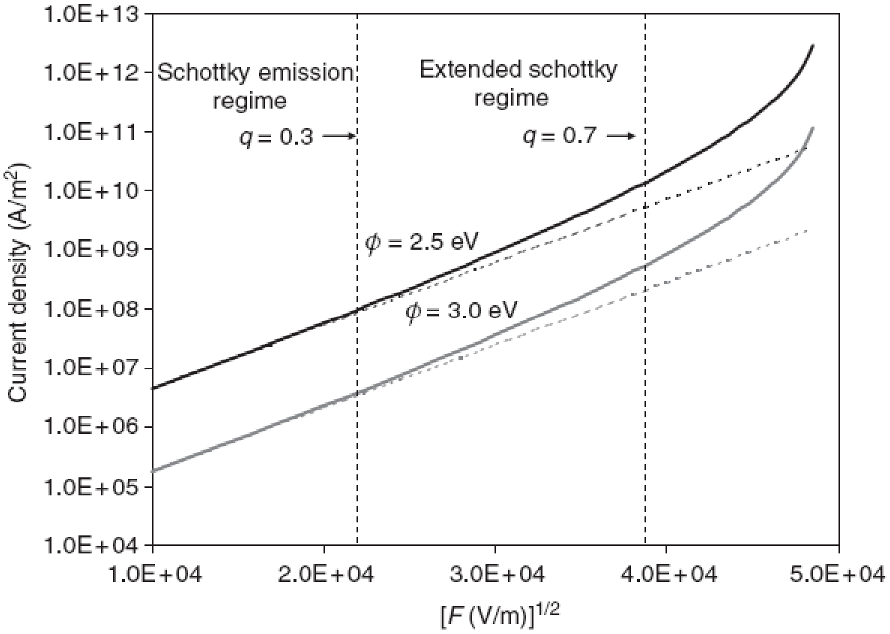

**Figure 1 - A plot of the analytical equations $J_{ES}$ (bold lines), and $J_S$ (dashed lines). The Schottky and Extended Schottky regimes are indicated.**

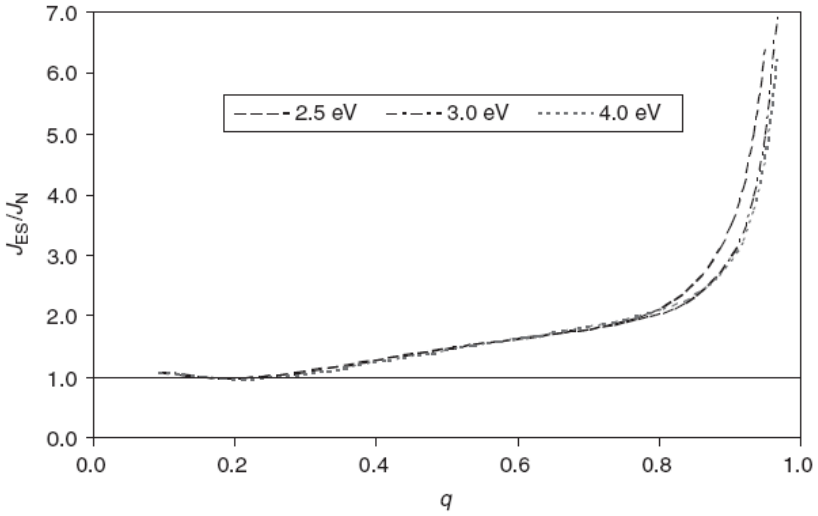

**Figure 2 - A plot of the ratio of the analytic equation $J$ to the numerical solution $J_N$ in terms of $q$ for three different work functions. A value of one shows agreement between the analytic equation and the numerical simulation.**

## Works Cited

[1]Swanson, L. S. and Schwind, G. A. (2008). Review of ZrO/W Schottky Cathode. In J. Orloff, *Handbook of Charged Particle Optics, Second Edition* (pp. 5-6). New York: CRC.

[2]Bahm, A., Schwind, G. A. and Swanson, L. W. (2008). Range of validity of field emission equations. *JVST* , 2080-2084.