There is plenty of room … in more dimensions

A simple harmonic oscillator (SHO) is one of the first problems introduced in a university Physics class and can be described by a 2nd order differential equation that has a simple, analytical solution: a sinusoidal response with a known period and amplitude. Perturbations of the SHO problem, adding a driving term, a damping term and/or a nonlinear spring constant, have periodic, deterministic solutions. A physical example of a SHO is a clamped-clamped beam, which can easily be fabricated using micromachining techniques. The expected response of the MEMS beam would seem to be very mundane. However, in our current work, we show that a simple clamped-clamped MEMS resonator, being driven by a single frequency tone, can exhibit both long term transient responses and complex, robust long-term repetitive responses. This is accomplished by allowing multiple modes of the simple beam to interact with each other, thus introducing more dimensions to the 2nd order differential equation through an internal mode coupling term. The interaction between modes creates bifurcations in the resonator response that yield more complex responses. A saddle node bifurcation is responsible for the long term transient response. The transient response is activated by the application of a short increase in the drive voltage (Fig. 1 red curve) that changes the response of the resonator. Following this drive amplitude pulse, the resonator moves away from a stationary operating point in phase space via an excursion and eventually returns to the stationary point (Fig. 1 blue curve). The transient response is robust and the resonator returns to the stationary operating point after each pulse. The timescale of this excursion is on the order of thousands of individual oscillations of the resonator. Other bifurcation structures can be encountered at different operating voltages and frequencies. At a slightly higher driving frequency, a saddle node on an invariant circle (SNIC) bifurcation creates long-term periodic behaviors on the same timescale as the transient response. Near the SNIC, the resonator does not have a stationary operating point, rather, the resonator moves in a specific trajectory in phase space, however, periodically encountering the remnant of the bifurcation. As the resonator moves along in phase space, it encounters a saddle point and ends up on one of the two excursions from the saddle point before returning back to the original trajectory. If left unperturbed, the resonator will continuously cycle through this response, randomly ending up on either of the two excursions (Fig. 2). However, the resonator can deterministically be controlled to end up on either of the excursions through the application of a small stimulus occurring at the correct time. As an example, the resonator can be forced to alternately execute the two excursions (Fig. 3).

Through characterization of the MEMS resonator, we can determine all the coefficients in the mathematical model and show high fidelity between the results of the numerical simulations and the experimentally measured resonator response. Using the numerical results, we can create a controllable complex response system. Additionally, the resonator can serve as a model for other complex, dynamical systems with similar mathematical models such as excitable or bursting neurons. The resonator can offer a physical platform for understanding how a system can be both robust and adaptable while incorporating physical factors that aren't always captured by mathematical models. Going forward, we plan to explore the complex responses of MEMS resonators that are "hidden" in higher dimensions and ways to control and use them for greater understanding of similar dynamical systems.

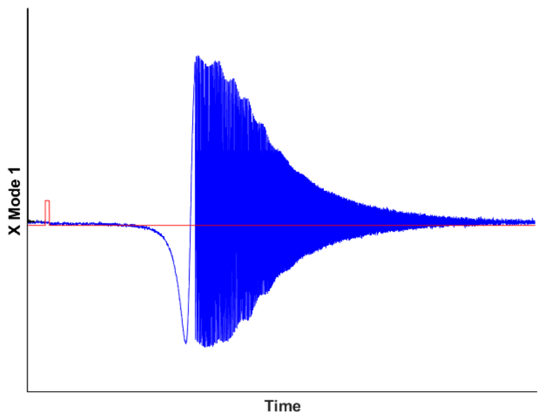

Figure 1 Graph showing a transient response of the resonator (blue) to a short stimulus pulse (red). After the transient response, the resonator returns to its original operating position in phase space. Repeated stimuli show the same response.

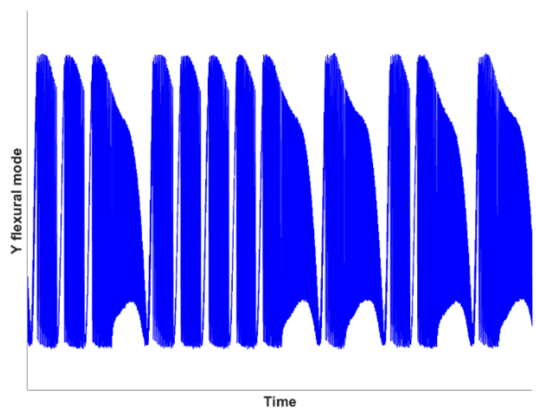

Figure 2 Graph showing the two excursions of the resonator resulting from the saddle point. If left unperturbed, the resonator randomly moves through both excursions.

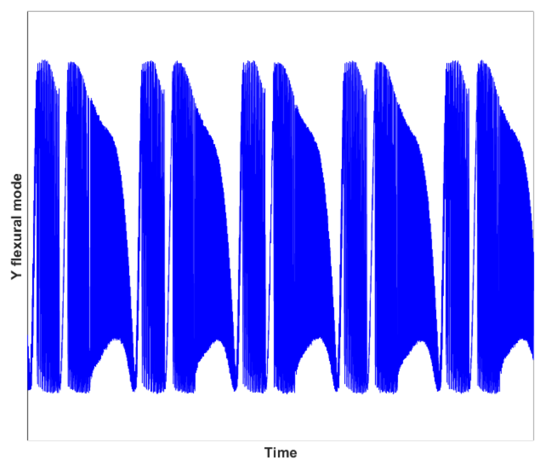

Figure 3 Graph showing the response of the resonator to the same operating conditions as in Fig. 2 with a short stimulus applied to the resonator at the correct time, forcing specific excursions. In this case, alternating excursions are shown.