Memristor-based Analog Optimization Solver for Safety-critical Control

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We introduce a new architecture for implementing safety-critical controllers on small-scale embedded systems, utilizing a hybrid of analog and digital computing to significantly lighten the computational demands. In our system, digital computing is employed for basic arithmetic, while complex safety-critical quadratic programming is handled by innovative analog technology, realized through custom-designed memristor devices. These devices are pivotal in dynamically tuning control-barrier-function-based (CBF) safety constraints, and we've developed a method for efficient closed-loop conductance tuning of these memristors. The practical effectiveness of our hybrid computing approach is showcased in an integrated prototype circuit within a flight control system. In tests involving repeated collision avoidance maneuvers with a micro-quadrotor UAV, our system not only successfully avoided collisions but also achieved a significant reduction in processing time by an order of magnitude.

Figure 1a depicts an optimization circuit which translates a constrained linear programming (LP) or quadratic programming (QP) problems for solving. Upon reaching a stable state, the solution to the problem is represented by the voltages across each column. Detailed validation of this circuit's functionality has been previously established in research [1]. Using memristors in the cross points we were able to use this optimization circuit in real-time to facilitate CBF based collision avoidance. Figure 1b shows I-V characteristics of our in-house fabricated memristors for 0.5mA, 1mA and 1.5mA compliance currents. As seen in the figure, the compliance current governs the achieved conductance of the memristors.

Figure 2a the PCB version of our Analog QP Solver. At each timestep, this circuit is used to solve optimization problem in real-time and send the solution to a micro-drone (Crazyflie 2.1) over wi-fi. The circuit system includes memristor tuning circuit, analog optimization solver circuit and communication components. Following the optimization problem solutions from the circuit, the drone can successfully avoid obstacles in its trajectory (Figure 2b).

Figure 3 shows the performance parameters of our Analog QP Solver. Over 4500 QP problems were solved using the circuit and the solutions were compared with the digital solution from MATLAB. The error found in the solution is depicted in Figure 3a. The median of the error was found to be 0.62%. Figure 3b shows the solve time of our QP solver. A step response was applied as the input and solving time was found to be around 10µs on average, whereas the digital solver takes over 3ms on average to solve a similar QP problem.

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[1] Vichik, S., & Borrelli, F. (2014). Solving linear and quadratic programs with an analog circuit. Computers & Chemical Engineering, 70, 160-171.

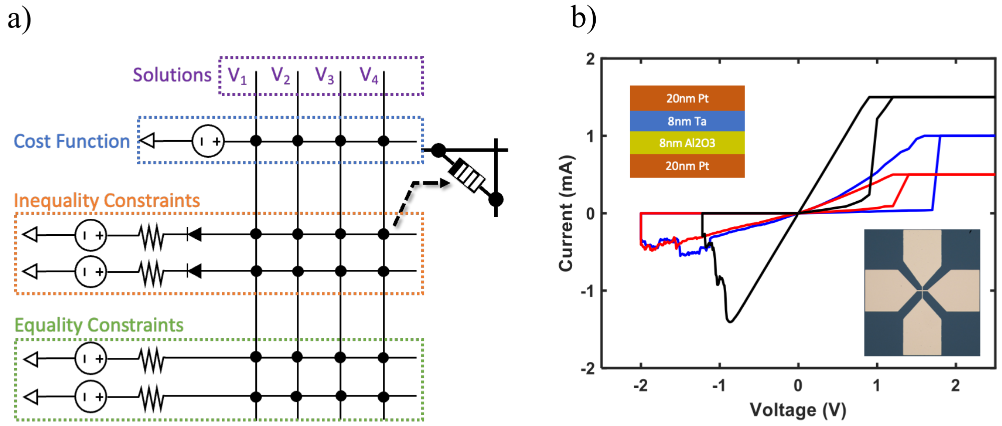

Figure 1: **a)** Simplified system architecture of a memristor-based QP Solver **b)** Memristor I-V characteristics for three different compliance currents, i.e., 0.5mA, 1mA and 1.5mA.

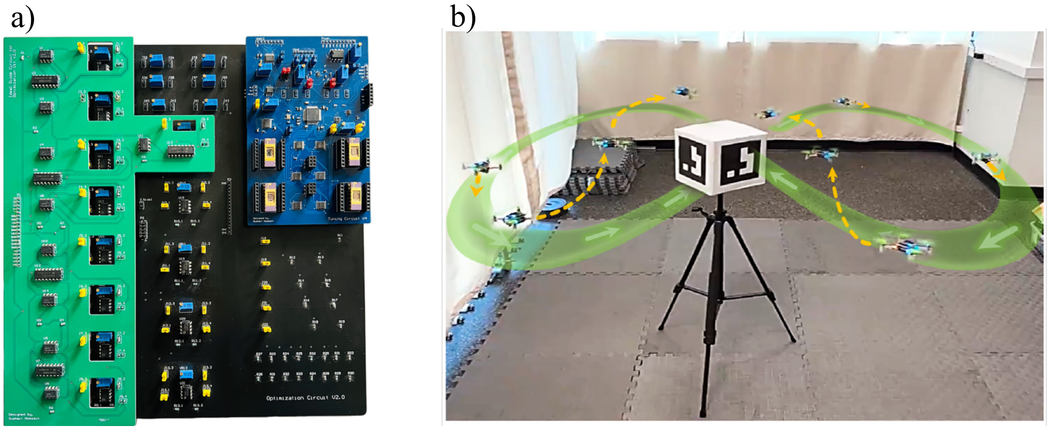

Figure 2: **a)** Analog QP solver circuit. **b)** Flight trajectory of the micro-UAV showing collision avoidance.

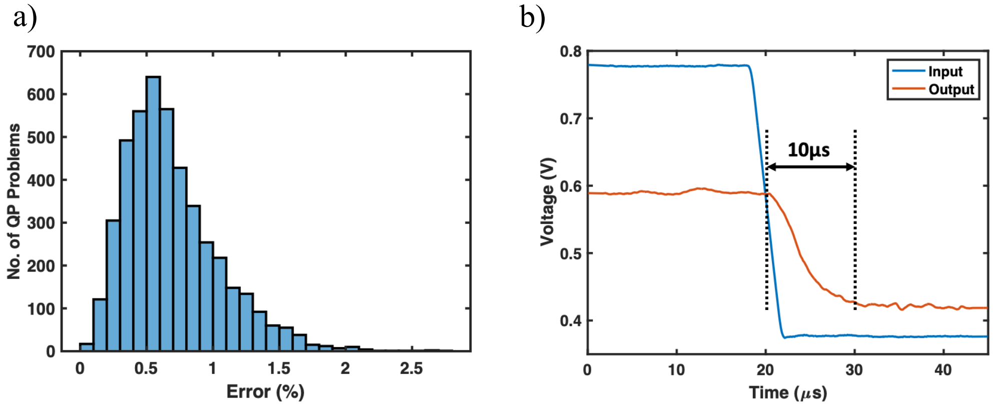

Figure 3: a) Analog QP Solver error (%). (b) Analog QP Solver latency.