Application-Specific Fast Multipole Methods for Enhancing Computational Efficiency in Curvilinear Mask Pattern Prediction

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With the continuous shrinking of feature sizes and the introduction of curvilinear (CL) pattern in advanced mask manufacturing, the computational overhead for pattern prediction (PP) has grown substantially. Conventional (conv.) PP typically relies on 2D-FFT to enhance efficiency; however, as pattern complexity and precision demands increase, 2D-FFT faces a significant computational bottleneck. While the conv. fast multipole method[1] (FMM) serves as one potential alternative, its computational time becomes excessive under high pattern density (PD). To tackle this, we propose ASFMM and its singular value decomposition enhanced version (ASFMM+SVD), significantly improving computation efficiency for high PD patterns. Besides, this is the first paper to apply FMM on CL pattern prediction. Fig. 1(a) illustrates the multi-level quadrant division: a quadrant is subdivided into four sub-quadrants if its non-zero dose pixels exceed a predefined value. Each sub-quadrant is treated as a node, with nodes of the same size stored at the same level. Fig. 1(b) depicts the conv. FMM D-to-M process, where a node accumulates dose contributions into multiple multipoles (e.g., 16 multipoles). In contrast, in Fig. 1 (c) we reduce the multipole numbers to one to increase computational efficiency. Although this introduces minor approximation errors, the results demonstrate that its impact on accuracy is insignificant. Fig. 1(d) shows a sharp decline in PSF intensity in the near-field and negligible slope variation in the far-field, justifying the use of multipole approximation to reduce runtime with minimal error. Fig. 1(e) depicts the conv. FMM workflow, which involves expensive multipole-to-local (M2L) translations and a downward pass that must propagate contributions to all child nodes. In contrast, Fig. 1(f) shows that we perform scalar matrix multiplication between the accumulated dose of the node and its corresponding far-field. Furthermore, in the enhanced ASFMM+SVD version, SVD is employed to optimize the near-field convolution for additional efficiency. To evaluate the proposed methods, we (1) calculated 50keV acceleration voltage point spread function for High-NA EUV mask, and (2) compared the fidelity and runtime against the conv. 2D-FFT method. Fig. 2(a) shows that the contours of ASFMM and ASFMM+SVD on the CL via layer are highly consistent with those of 2D-FFT. Efficiency comparisons in Fig. 2(b) and (c) demonstrate that proposed methods achieve linear scalability as pattern size increases and remain unaffected by PD, overcoming a major disadvantage of conv. FMM. Table I demonstrates that the proposed methods significantly reduce runtime while maintaining negligible EPE_{mean} and NMSE. ASFMM+SVD stands out as the most efficient, achieving a 93% runtime improvement and a 1,411% speed-up ratio compared to the 2D-FFT method. In conclusion, the proposed ASFMM and ASFMM+SVD methods effectively overcome efficiency degradation in high PD patterns, offering exceptional scalability and runtime performance for PP.

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[1] W. Yao et al., *IEEE Trans. Comput.-Aided Des. Integr. Circuits Syst.*, **42**, 1 (2023).

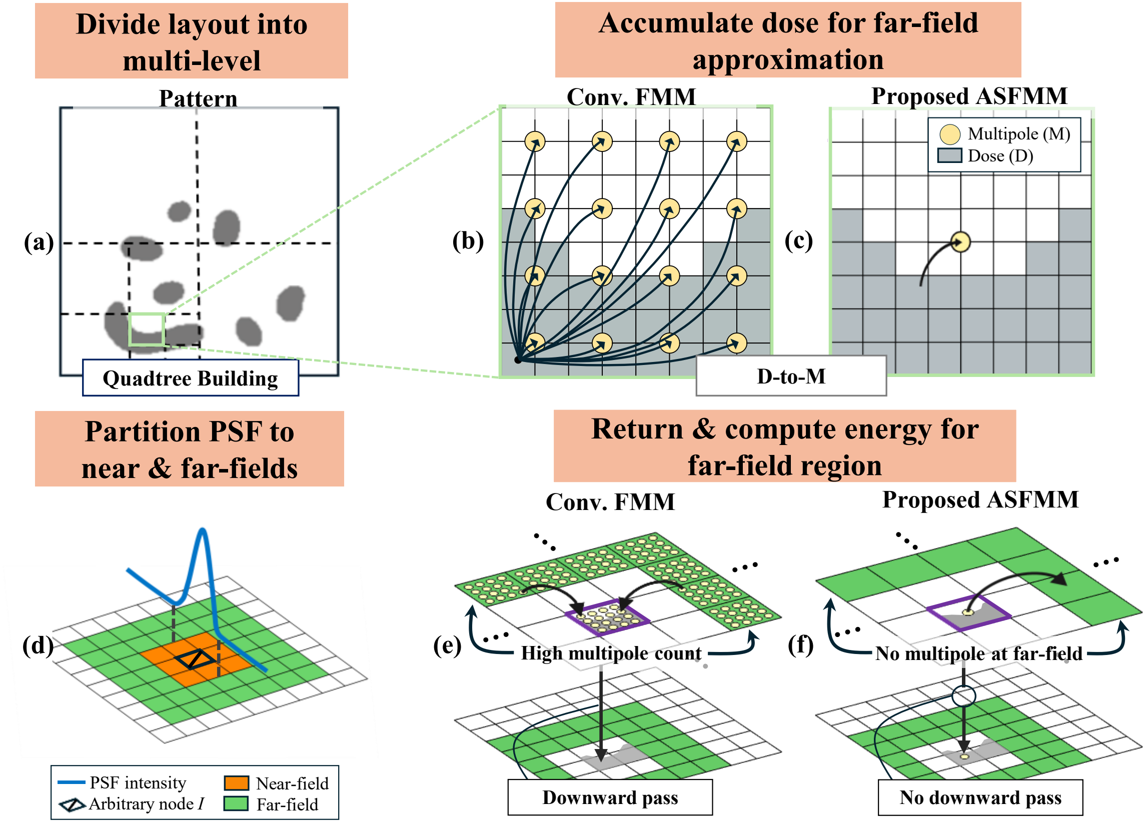

Fig. 1 Illustrations of the conventional FMM and proposed ASFMM method. (a) Pattern is divided as quadtree. (b) The FMM computes the contribution of each dose to every multipole during the D-to-M process. (c) Proposed D-to-M process. (d) PSF intensity distribution across near-field and far-field regions for node *I*. (e) Nodes of different sizes exhibit varying far-field ranges; the FMM computes contributions from the multipoles of far-field nodes to the target node. (f) The proposed methods compute the interaction between the multipole's weight and it's corresponding far-field and eliminate downward pass.

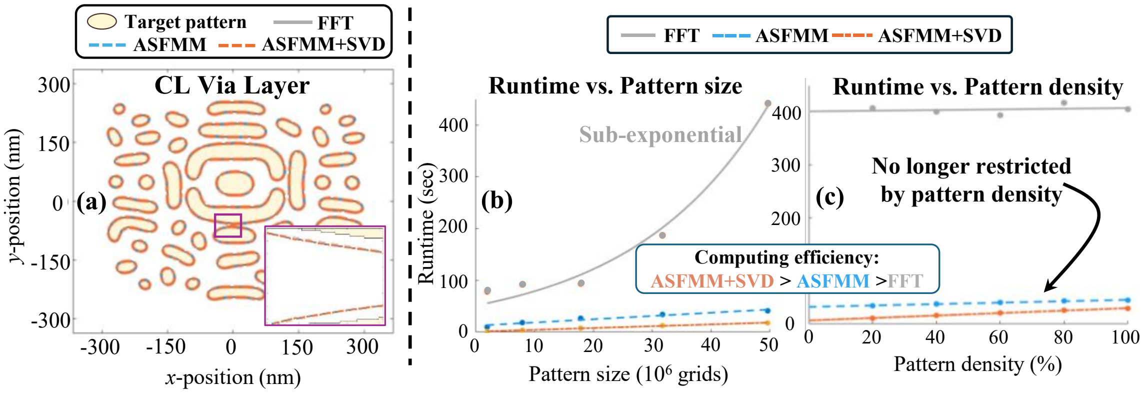

Fig. 2 Indicate the accuracy and efficiency of conventional and proposed methods. (a) Contours for curvilinear via layer. (b) Runtime versus pattern size. (c) Runtime versus pattern density.

Table I Numerical comparison of accuracy and efficiency between the conventional and proposed methods.

Methods		Accuracy				Efficiency		
		NMSE (%)	EPE_{mean} (nm)	EPE_{max} (nm)	EPE_{std} (nm)	Runtime (sec)	Runtime Improv.	Speed-up Ratio
Conv.	**2D-FFT**	—	—	—	—	**307.80**	—	—
Proposed	**ASFMM**	**0.54**	**0.0579**	**0.3606**	**0.0539**	**37.93**	**88%**	**811%**
	ASFMM +SVD	**0.54**	**0.0579**	**0.3606**	**0.0539**	**21.81**	**93%**	**1,411%**

Negligible error (smaller than 0.1nm GS) **Significantly improved**